(NASA-CR-146027) LARGE EDDY STRUCTURE OF A TURBULENT BOUNDARY LAYER Semiannual Progress Report (Texas Univ.) 63 p

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Semi-Annual Progress Report
(May-November 1975)

on

NASA Ames Grant NSG-2077 Entitled

"Large Eddy Structure of a Turbulent Boundary Layer"

to

Dr. Morris W. Rubesin
Senior Staff Scientist and
Grant Technical Monitor
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by

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ABSTRACT

Current results include digital codes for the first half of a four-phased program to extract eddy structures from experimental, temporally averaged, two-point, velocity correlations. Merger of the two major data sources, Grant (1958) and Tritton (1967), is complete and has yielded a successful relaxation method for extending the data grid to off-axes probe separations. Preliminary results indicate an eddy structure elongated in the flow direction but it is too early to comment upon the common presumption of "horse-shoe" eddies in the wall layer. To recapitulate, the four phases are:

I. Augmentation of Data Grid of Correlation Tensor

II. Fourier Transform in Planes Parallel to the Wall

III. Extraction of the Eigen-modes ("Large Eddies")

IV. Construction of B_{ij}, the "Large Eddy" Correlation Tensor

Phase II greatly reduces the calculational complexity. Phase III relies upon Lumley's (1964) Proper Orthogonal Decomposition Theorem ("PODT"). Phase IV impacts heavily upon the "sub-grid" modelling technique for turbulent flow calculations. A major conclusion of the work should be a priori justification (or not!) for the validity of the spatially homogeneous, even isotropic, "eddy viscosity" approach for the small-scale turbulence. If the result is affirmative, then many "ad hoc" calculational schemes will be placed upon a more rational basis. Intutitively, one expects such an answer, particularly in view of previous work at Penn State in this area (experimental by Bakewell, 1966; semi-empiric and analytic by Payne, 1966, 1967, and 1968a).

CONTENTS:

I. Brief Historical Survey and Introduction:

A. A. Townsend (1956)* laid the fundamental basis for this work by postulating a "two-component" model of turbulence based upon the actions of "Large Eddies" to increase greatly transports of fluid properties; these large eddies "feed" efficiently upon the mean flow and, in turn, "feed" the rest of the turbulence. Lumley (1964) was the first to give a rational definition of these structures. His students extracted these structures from the 2-D wake (Payne, 1966, 1967) and measured them in the wall layer (Bakewll, 1966). By a non-linear analysis of the disturbance kinetic energy equation Lumley (1966) provided a possible scheme for prediction of the "Large Eddies" which had some success in the wall-layer (Elswick, 1967) and even more success for the 2-D wake (Payne, 1968a).

This approach remained virtually in hiatus until NSG-2077 expecting occasional works by Payne (1968b, 1969a,b, 1973a,b, 1974a,b, 1975a) none of which attacked the boundary-layer problem directly. For more historical detail see Payne 1975b, enclosed as Appendix, which is a copy of slides used in a seminar given at NASA Ames November 7, 1975.

In brief, the approach funded by NASA Ames Grant NSG-2077 is to use all available experimental two-point, temporally averaged, velocity covariance data for the flat-plate boundary-layer and to extract from those data the PODT** eigen-modes which Lumley (1964) defined and interpreted as the "Large Eddies."

Motivation for extraction of these structures is, finally, to

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*See References, Page 7-8

**PODT = Proper Orthogonal Decomposition Theorem calculate B_{ij}, the "Large Eddy" co-variance tensor, and remove these components from the full correlation tensor, R_{ij}. Then, one can calculate an eddy viscosity for the rest of the turbulence, namely, the "small eddies," via

$$\nu_e \equiv (B_{ij} - R_{ij})\left[\frac{\partial U_1}{\partial X_2}\right]^{-1} \tag{1}$$

where (1) is a generalization of the usual definition:

$$\nu_e \equiv -\overline{UV}\left[\frac{\partial U_i}{\partial X_j}\right]^{-1} = -R_{12}\left[\frac{\partial U_1}{\partial X_2}\right]^{-1} \tag{2}$$

wherein we put $i = 1$, $j = 2$ from equation (1) and do <u>not</u> extract the large eddy contribution from the Reynolds' stress. (See Appendix p 17,22 and 26)

II. Results to Date:

A. Merging of Data Sets:

The only sets of useful data which provide spatial correlations of the Reynolds' stress tensor are those of Grant (1958) and Tritton (1967). These use differing normalizations but can be merged using Townsend's (1956) intensity measurements (Appendix, p. 27, 28, 39).

This difference arose from Tritton's use of uncalibrated probes via

$$\tilde{R}_{ij} = \overline{U_i(\underline{X})U_j(\underline{X}')}\,[\overline{U_i^2(\underline{X})}\,\overline{U_j^2(\underline{X}')}]^{-1/2} \tag{3}$$

Whereas Grant used calibrated probes:

$$\tilde{R}_{ij} = \overline{U_i(\underline{X})U_j(\underline{X}')}\,[\overline{U_i^2(\underline{X})}\,\overline{U_j^2(\underline{X})}]^{-1/2} \tag{4}$$

i.e., it is required to know the "denormalizing" factor $[\;]^{-1/2}$ in equations (3) and (4). Townsend's monograph provides this information.

B. Methods Considered:

Pre-NASA Grant work (1974-1975) by Payne and Lemmerman showed that Payne's (1969a) curve-fitting approach could be bypassed by direct, digital Fourier Transform of R_{ij} in the homogeneous planes (downstream and cross-stream) parallel to the wall. It should be noted that Payne's original 3-D work (1966) was performed on an IBM 7074 whereas a CDC 6600 is available for this work--a factor of 100 in core and speed does simplify!

There remains the question: "How to extend Grant's/Tritton's data to off-axes separations?" If the problem were 1-D, a parabolic interpolation would be reasonable. Extending this concept to 3-D, since we have data on the $\underline{r}$-axes ($\underline{r} = \underline{x}' - \underline{x}$) and know that R_{ij} must vanish as $\underline{r} \to \infty$, a relaxation method appears appropriate. Hence, the Laplace (and Poisson) equation is used to "fill-in" the off-axes R_{ij} data. (See Appendix, p. 29, 30)

C. Results to Date:

The coding for full, 3-D relaxation methods to "fill-in" off-axes R_{ij} is complete. See Appendix A, p. 31 to 35 for digital graphics results. These indicate that scales in the flow direction much exceed vertical and transverse scales. Of course, one can also see this from perusal of the 30-odd correlation curves of Grant, but it is far more graphic in 3-D plots than 2-D graphs. These plots (Appendix, p. 31 to 35) give a posteri validity to the chosen relaxation method for data augmentation.

Also complete is a 1-D Fourier transform computer program. Extension to (the final) 2-D form is virtually complete. The FFT* algorithm

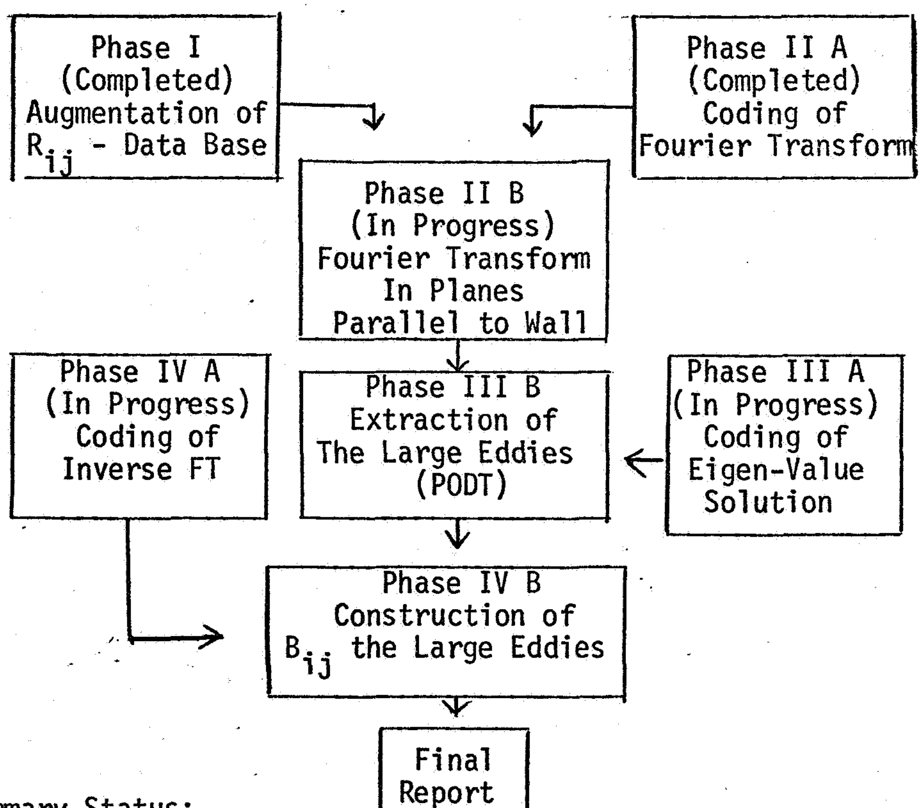
*FFT = Fast-Fourier Transform

is not being used due to certain accuracy problems. However, the FT+ code developed has been verified by test cases into Fourier-Space and inverse FT; accuracy is within 1% at all points which is more than adequate for experimental data which are typically of 5-10% accuracy.

III. Outline of Remaining Effort:

As indicated during the November 1975 presentation to NASA Ames, the major task is establishment of a data access "tree" for the computer (Appendix A, p. 44). The remaining efforts will proceed via three parallel paths as indicated schematically below:

Schematic of NSG-2077 Task Status
(As of November 30, 1975)



Summary Status:
1. Completed = Phase I, II A
2. In Progress = Phase II B, III A, IV A
3. Planning Complete = Phase III B, IV B

+FT = Fourier Transform

IV. Closure and Summary:

The rather massive data management problem on the digital computer is under control and planning is complete for the remaining tasks. It will be most interesting to see whether the calculated eigen-values and vectors (via PODT) fall off slowly in amplitude (like the 2-D Wake, Payne 1966, 1967) or rapidly (like the wall eddies, Bakewell, 1966). The later will probably be more nearly the case for the boundary-layer which is, of course, to be hoped for if "sub-grid" modelling is to be tractable. If the former is the case (which means <u>many</u> modes are contributing to the "Large Eddy") then "ad hoc" closure to the turbulence problem may have to be carried to considerably higher order than second.

It should be noted that, since the data of Grant and Tritton extend only to about 1/2 δ_{99}, the structures extracted herein include the wall region (viscous sub-layer and logarithmic) and only part of the "wake" region of the flat-plate boundary-layer. To completely delineate the entire structure of a boundary layer would require either: a) new experiments or b) some sort of "patching" of NSG-2077 results with a "wake" component. Possibly, depending upon what NSG-2077 eddies look like, one might be able to incorporate the 2-D wake "large eddy" (Payne, 1966, 1967, 1968a) results. A decision <u>must</u> await final results of this grant.

Hence, the implication of this work for eventual digital "experimentation" (i.e., simulation of turbulence) via Navier-Stokes, Reynolds', or higher order "closure" equations appear to be quite crucial for the simplest proto-type boundary-layer, namely that of a flat-plate at zero incidence neglecting compressibility effects.

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APPENDIX

"The Large-Scale Structure of Turbulent-Shear-Flows"
(Vu-graphs of Seminar to NASA Ames, 7 November 1975)

CONTENTS

ERRATA

To NSG-2077 Oral Progress Report, November 7, 1975

Page	Item	Should Read
2	Grant (1959)	Grant (1958) (Two places)
2	Tritton's (1966)	Tritton's (1967)
8	$\frac{\partial U_i}{\partial U_j}$	$\frac{\partial U_i}{\partial x_j}$ (line 7)
10	L.H.S. Eq. (6)	$\int R_{ij}(\underline{x};t;\underline{x}',t')\,\phi_j(\underline{x}',t')\,d\underline{x}'dt'$
15	Reference omitted	(Payne, 1966)
19	$e^{i\underline{k}\cdot\underline{r}}$	$e^{i\underline{k}\cdot\underline{x}}$ (line 2)
20	Eq. C1, RHS	$\lambda^{(n)}(\underline{k})\psi_i^{(n)}(\underline{k},y)$
22	Phase (VI)	Phase (IV)
21	Grant's (1959)	Grant's (1958) (line 14)
38	$u_i(\overline{x})u_j(\overline{x}')$	$\overline{u_i(\overline{x})u_j(\overline{x}')}$ (6 occurrences)
39	$u_i(\overline{x})u_j(\overline{x}')$	$\overline{u_i(\overline{x})u_j(\overline{x}')}$ (2 occurrences)
48.	Grant, H.L. (1959)	Grant, H.L. (1958) (line 5)

THE LARGE-SCALE STRUCTURE OF TURBULENT - SHEAR - FLOWS

AN HISTORICAL REVIEW AND SEMI-ANNUAL PROGRESS REPORT

TO

DR. MORRIS W. RUBESIN, GRANT MONITOR
SENIOR STAFF SCIENTIST, NASA/AMES

ON

NASA RESEARCH GRANT NSG-2077
"LARGE EDDY STRUCTURE OF A TURBULENT BOUNDARY LAYER"

BY

F.R. PAYNE AND L.A. LEMMERMAN
THE UNIVERSITY OF TEXAS AT ARLINGTON

NOVEMBER 7, 1975

ABSTRACT

A BRIEF SURVEY OF TURBULENT FLOW CHARACTERISTICS IS FOLLOWED BY A DETAILED OUTLINE OF PRIOR WORK INVESTIGATING THE "STRUCTURE" ("EDDIES") OF REAL FLUID SHEAR FLOWS. THESE EFFORTS ORIGINATE IN TOWNSEND'S (1956) POSTULATE OF A "DOUBLE STRUCTURE" ("LARGE" AND "SMALL" EDDIES) IN TURBULENCE. UNTIL LUMLEY (1965) THERE EXISTED NO RATIONAL WAY TO DEFINE THESE "LARGE EDDIES" OR EVEN TO EXTRACT THESE STRUCTURES FROM EMPIRICAL DATA.

BASED UPON LUMLEY'S WORK, HIS STUDENTS WERE SUCCESSFUL IN EXTRACTING FROM EXPERIMENT THE LARGE EDDY STRUCTURE FROM THE FOLLOWING FLOW PROTO -TYPES: 2-D WAKE OF A CIRCULAR CYLINDER (PAYNE, 1966). PARTIAL SUCCESS WAS OBTAINED IN PREDICTING, FROM FIRST PRINCIPLES, THESE EDDIES IN: VISCOUS SUB-LAYER (ELSWICK, 1967) AND 2-D WAKE (PAYNE, 1968).

THIS NASA GRANT IS SOLEY CONCERNED WITH EXTRACTION OF THESE STRUCTURES FROM FLAT PLATE BOUNDARY LAYER EXPERIMENTS. THE PROCEDURES USED ARE THOSE SUCCESSFUL IN THE 2-D WAKE IN 1966. THE GRANT EFFORT IS NEAR THE HALF-WAY POINT OF TOTAL EFFORT EVEN THOUGH THE FIRST TWO OF FOUR PHASES ARE NOT QUITE FINISHED.

OVERVIEW OF REPORT

I. INTRODUCTION:

A. MOTIVATION - WHY DO THE PROBLEM?

B. PRIOR WORK

1) TOWNSEND (1956) } CONFLICTING MODELS OF "LARGE EDDIES"

2) GRANT (1959) }

3) LUMLEY (1965) - PODT - DEFINED "LARGE EDDY"

4) LUMLEY (1966) - ORR - PREDICTION METHOD

5) RESULTS (1966-1968) - 2-D WAKE; WALL LAYER

C. STATEMENT OF CURRENT PROBLEM

II. REVIEW OF LUMLEY'S EXTRACTIVE METHOD (PODT) AS APPLIED TO THE FLAT-PLATE BOUNDARY LAYER.

A. SIMPLIFICATION DUE TO HOMOGENEOUS FLOW STATISTICS

B. IDENTIFICATION OF THE "BIG EDDIES"

C. RECONSTITUTION OF (STATISTICALLY PRESISTENT)FLOW VELOCITIES

D. RECAPITULATION OF SEQUENTIAL STEPS IN THE METHOD

III. MAJOR EFFORTS OF NASA GRANT NSG - 2077

A. MERGING OF GRANT'S (1959) AND TRITTON'S (1966) DATA AND METHODS TO FILL DATA VOIDS

B. FOURIER TRANSFORM OF VELOCITY CO-VARIANCE TENSOR

C. EIGENVALUE SOLUTION IN FOURIER SPACE

D. INVERSE FOURIER TRANSFORM TO RECONSTRUCT VELOCITIES AND EDDY VISCOSITY

IV. SUMMARY STATUS OF RESULTS AND BUDGET

V. WHAT WILL REMAIN AFTER CURRENT WORK COMPLETED?

A. SHORT TERM

B. LONG TERM

TURBULENCE IS:

1. RANDOM ("STOCHASTIC") ---- MUST USE STATISTICS
2. ROTATIONAL ---- NO VELOCITY POTENTIAL
3. DIFFUSIVE ---- "BETTER MIXER"
4. DISSIPATIVE ---- FLOW LOSSES INCREASED
5. FULLY 3-D ---- $\sim 10^{24}$ STORAGE LOCATIONS
6. NON-LINEAR ---- TOUGH! CAN'T LINEARIZE
7. CONTINUUM ---- NAVIER-STOKES EQ. GOVERNS

- NAVIER-STOKES EQUATIONS: (INCOMPRESSIBLE, $\nabla \cdot \underline{V} = 0$)

$$\frac{\partial \underline{V}}{\partial t} + \underline{V} \cdot (\nabla \underline{V}) = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \underline{V} \tag{1}$$

$\underline{V}$	=	VECTOR VELOCITY
ρ	=	MASS DENSITY
P	=	PRESSURE
ν	=	KINEMATIC VISCOSITY
∇	=	"DEL" OPERATION

I. INTRODUCTION

A. MOTIVATION:

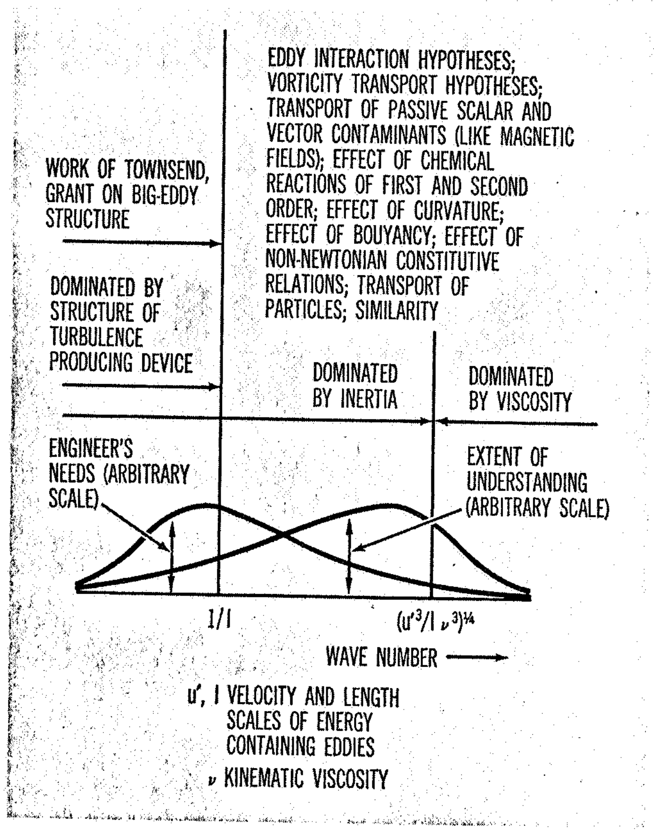
"BIG WHIRLS HAVE LITTLE WHIRLS THAT FEED ON THEIR VELOCITY; LITTLE WHIRLS HAVE LESSER WHIRLS AND SO ON TO VISCOSITY" -- RICHARDSON (?) (CIRCA 1920)

o BIG WHIRLS ≡ LARGE-SCALE-STRUCTURE ("LARGE EDDIES")

1) WHY DO INCOMPRESSIBLE TURBULENCE?
   A. IF TURBULENT-MACH NO.<<1, LITTLE DIFFERENCE ($M_{00} \leq 5 \Rightarrow M' \leq 0.2$, HINZE, 1975)
   B. EVEN INCOMPRESSIBLE TURBULENCE NOT WELL UNDERSTOOD.
2) AS NEXT SLIDE SHOWS, ENGINEER'S NEEDS VARY INVERSELY WITH AVAILABLE TURBULENCE KNOWLEDGE.
3) WHY DO "BIG EDDIES"? - PRESUMABLY THEY DETERMINE THE VASTLY INCREASED TRANSPORT (OVER MOLECULAR DIFFUSION) OF:

   MASS ($\delta \sim x$ RATHER THAN $\sqrt{x}$)
   MOMENTUM ($\nu_e \sim 10^2 \nu$)
   HEAT ($k_e \sim 10^2 k$)

   AND BIG EDDIES CANNOT BE LINEARIZED
   (OR "BABY THROWN OUT WITH HIS BATH WATER")



Comparison of understanding of turbulence to Design Engineer's needs (after Lumley, 1967)

4) POTENTIAL PAYOFFS (OF "BIG EDDY" KNOWLEDGE)

FLOW PROTO-TYPE	DESIGN PAYOFF
BOUNDARY-LAYER	1. FOREBODY/FUSELAGE/DUCT
	2. INLET RAMP/SPIKE
	3. LIFTING SURFACES
	4. VORTEX GENERATORS
	5. HI-LIFT & AUGMENTATION
3-D WAKE	1. VORTEX GENERATORS
(2-D NEAR WAKE)	2. FLOW STRAIGHTENERS/VANES
	3. DUCT CRUCIFORMS
	4. TURBULENCE GENERATORS FOR MODEL TEST
	5. AB FLAME HOLDER
	6. AFTERBODY DESIGN
JET	1. EXHAUT/FUEL NOZZLE
	2. COMBUSTER (?)
	3. NOISE SUPPRESSION
	4. IR SUPPRESSION (?)
	5. TEST GENERATORS
GRID	1. FLOW MIXERS
	2. IR/RADAR ATTENUATION
	3. GUIDE VANES

MIXING LAYER

1. NOZZLE
2. SECONDARY DUCT/NOZZLE
3. COMBUSTER (?)
4. VTOL

I. B. PRIOR WORK:

1) <u>1956 - TOWNSEND</u> FORMULATED THE "DOUBLE STRUCTURE" MODEL OF TURBULENCE: (SEE PAYNE, 1966)

TOWNSEND: A. FULLY TURBULENT FLUID BOUNDED BY CONTORTED SURFACE MOVED ABOUT BY "LARGE EDDIES" ("LARGE" = WIDTH OF FLOW WHERE $\frac{\partial U_i}{\partial U_j}$ SIGN SAME)

B. OUTSIDE IS <u>NOT</u> TURBULENT AND $\underline{\omega} = 0$

C. EXCEPT NEAR "EDGES" TURBULENCE $\sim$ UNIFORM INTENSITY

D. "REST" OF TURBULENCE "FEEDS" ON THE BIG EDDIES AND "EQUILIBRIUM" OF ENERGY EXCHANGE EXISTS FOR THE 2-D WAKE

D) => BIG EDDY IS

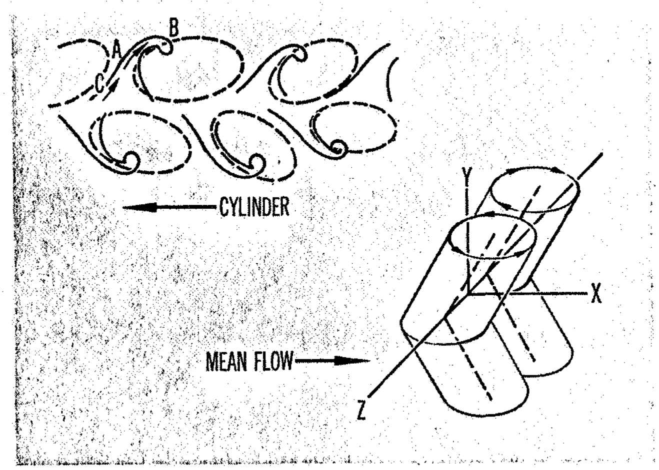
$$U = -V = AZ \text{ EXP}\left\{-\frac{\alpha^2}{2}(Y - Y_0)^2 + Z^2\right\} \text{ EXP}\left\{-\frac{\beta^2}{2}X^2\right\}$$

$$W = A(Y - Y_0) \text{ EXP}\left\{-\frac{\alpha^2}{2}(Y - Y_0)^2 + Z^2\right\} \qquad (1)$$

2. 1959 - GRANT, HAVING MORE EMPIRIC DATA, POSTULATED

$$\left.\begin{aligned} U &= A(1 - \alpha^2 Z^2) \text{ EXP}\left\{-\tfrac{1}{2}\alpha^2(X^2 + Z^2)\right. \\ V &= 0 \\ W &= A\alpha^2 XZ \text{ EXP}\left\{-\tfrac{1}{2}\alpha^2(X^2 + Z^2)\right. \end{aligned}\right\} \qquad (2)$$

FOR SKETCH OF GRANT'S MODIFIED MODEL, SEE NEXT SLIDE



LARGE EDDY STRUCTURE OF THE 2-D TURBULENT WAKE (AFTER GRANT, 1959); THE TOTAL STRUCTURE IS A SUPERPOSITION OF THE TWO SHOWN.

NOTE: BOTH MEN SHOW CONSIDERABLE INSIGHT AND IMAGINATIVE MODELING BUT THEIR APPROACH IS FLAWED BY, ESSENTIALLY, "HAVING TO GUESS A MODEL, THEN ADJUST CONSTANTS TO FIT."

I. B. 3) 1965 - LUMLEY

CONTRADICTION $\left\{ \begin{array}{l} \text{TOWNSEND} \\ \text{GRANT} \end{array} \right\}$ MODELS $\rightarrow$ LUMLEY TO WRITE:

"ALTHOUGH THE DYNAMIC ROLE OF THESE EDDIES HAS BECOME CLEAR, NO OBJECTIVE DEFINITION HAS YET BEEN OFFERED. IN THE ABSENCE OF SUCH A DEFINITION THEY CANNOT BE MEASURED, SINCE THEY CANNOT BE SATISFACTORILY DISTINGUISHED FROM THE REST OF THE FIELD, NOR CAN THEIR STRUCTURE BE PREDICTED FROM FIRST PRINCIPLES. THE FOLLOWING IS AN ATTEMPT TO PROVIDE AN OBJECTIVE DEFINITION." (EMPHASIS ADDED)

* LUMLEY'S "PROPER ORTHOGONAL DECOMPOSITION THEOREM" (PODT) SUMMARY OF PODT

GIVEN: A RANDOM VECTOR FIELD, $u_i = u_i(\underline{x}, t)$ (3)

SELECT: A (DETERMINISTIC) CANDIDATE $\phi_i = \phi_i(\underline{x}, t)$ (4)

TEST: THE CANDIDATE BY PROJECTION OF u_i UPON ϕ_i; SINCE ONLY PARALLELISM IN HILBERT SPACE, NOT AMPLITUDE OF ϕ_i, IS OF INTEREST, WE FORM THE SCALAR PRODUCT:

$$\alpha = \int \phi_i^* u_i \, d\underline{x}dt \Big/ \left(\int \phi_j \phi_j^* \, d\underline{x}dt \right)^{1/2} \tag{5}$$

STATISTICS: OF α, SIGN IRRELEVANT, SO MAXIMIZE $\overline{|\alpha|^2}$

$$\longrightarrow \qquad \int R_{ij}(x, t; x', t') \phi_j (x, t') = \overline{|\alpha|^2} \, \phi_i(x, t) \tag{6}$$

WHERE R_{ij} IS THE VELOCITY COVARIANCE, $\overline{u_i(\underline{x}, t) u_j(\underline{x}', t')}$

APPLICATION OF KNOWN THEOREMS; HILBERT-SCHMIDT$\longrightarrow$ MERCER'S THEOREM, ETC. (SEE PAPER) AND RESULTS:

A. THERE ARE DENUMERABLE SOLUTIONS TO (6):

$$\int R_{ij}\ \phi_j^{(n)} d\underline{x}' d\underline{t}' = \lambda^{(n)} \phi_i^{(n)}(\underline{x},t) \tag{7}$$

B. $\phi_i^{(n)}$ CAN BE CHOSEN AS ORTHO-NORMAL:

$$\int \phi_i^{(p)} \phi_i^{(q)*} d\underline{x}\ d\underline{t} = \delta_{pq} \tag{8}$$

C. THE RANDOM VECTOR FIELD U_i CAN BE EXPANDED:

$$U_i(\underline{X},T) = \sum_n \alpha_n\ \phi_i^{(n)}(\underline{X},T) \tag{9}$$

$$\alpha_n = \int U_i \phi_i^{(n)*} d\underline{x} dt \tag{10}$$

D. THE ONLY STATISTICAL QUANTATIES IN (9) RHS ARE UNCORRELATED:

$$\overline{\alpha_n \alpha_m} = \lambda^{(n)} \delta_{nm} \tag{11}$$

E. R_{ij} MAY BE DECOMPOSED INTO A DOUBLE SERIES:

$$R_{ij} = \sum_n \lambda^{(n)} \phi_i^{(n)}(\underline{X},T) \phi_j^{*(n)}(\underline{X}',T') \tag{12}$$

WHERE THE SERIES IN BOTH UNIFORMLY AND ABSOLUTELY CONVERGENT

F. THE $\lambda^{(n)}$ ARE NON-NEGATIVE WITH FINITE SUM:

$$\lambda^{(n)} \geq 0, \sum_n \lambda^{(n)} < \infty \tag{13}$$

AND THE EXPANSIONS (9), (12) ARE OPTIMAL IN THE SENSE THAT TRUNCATION OF THE SERIES AT $n = N$, FINITE LEAVES THE LEAST POSSIBLE REMAINDER IN THE (DENUMERABLY) INFINITY OF NEGLECTED TERMS.

SUMMARY OF PODT

A. EIGENVALUE PROBLEM:

$$\int R_{ij}\phi_j^{(n)} D\underline{X}'DT' = \lambda^{(n)}\phi_i^{(n)}(\underline{X},T), \text{ WHERE} \qquad (7R)$$

B. DECOMPOSITION OF R_{ij}:

$$\overline{u_i(\underline{X},T)u_j(\underline{X}'T')} = R_{ij} = \sum_n \lambda^{(n)}\phi_i^{(n)}(\underline{X},T)\phi_j^{(n)*}(\underline{X},T)$$

AND (12R)

C. CONSTRUCTION OF $u_i(X,T)$:

$$u_i(X,T) = \sum_n \alpha^{(n)}\ \phi_i^{(n)}(\underline{X},T) \qquad (9R)$$

I. B. 4) 1966 LUMLEY ("ORR")

SINCE PODT INTO NAVIER-STOKES STILL REQUIRES SOLUTION OF NON-LINEAR EQUATIONS, LUMLEY (1966) REVIVED THE SO-CALLED "ORR ENERGY METHODS" OF PROFILE STATILITY ANALYSIS. GIVEN: CM, CLM, CKE, INCOMPRESSIBLE NEWTONIAN FLUID, THE AVERAGED "GLOBAL" DISTURBANCE ENERGY ($\bar{E}$) EQUATION IS

$$\frac{\partial}{\partial t}\int \bar{E}\, dV = -\rho\int\overline{u_i u_j}\ S_{ij}dV - \int\mu\ \overline{d_{ij}s_{ij}}\ dV \qquad (14)$$

WHERE $\overline{E} = 1/2\ \overline{u_i u_i}$, $S_{ij} = 1/2(\frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i})$, THE MEAN STRAIN RATE,

$$d_{ij} = \frac{\partial u_i}{\partial X_j}\ ;\ s_{ij} = 1/2\ (d_{ij} + d_{ji}),$$

FLUCTUATING DEFORMATION AND STRAIN RATES.

<u>ASSUME</u>: GLOBAL DISTURBANCE KINETIC ENERGY IN STATIONARY:

$$\frac{\partial}{\partial t} \int \overline{E}\ dV = 0$$

<u>MAXIMIZE</u>: THE SPATIAL VARIATION OF VISCOSITY:

$$S_{ij}U_j = \phi,_i + \frac{\partial}{\partial X_j} \left[\nu_T(\frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i})\right] \qquad (15)$$

WHERE "ϕ CAN BE INTERPRETED AS A PRESSURE" AND ν_T IS AN "EDDY VISCOSITY."

NOTE: EQ. 15, BEING LINEAR, IS, IN PRINCIPLE, EASILY SOLVED.

I. B. 5)

COMPARISON/RESULTS OF PODT/ORR THROUGH 1974

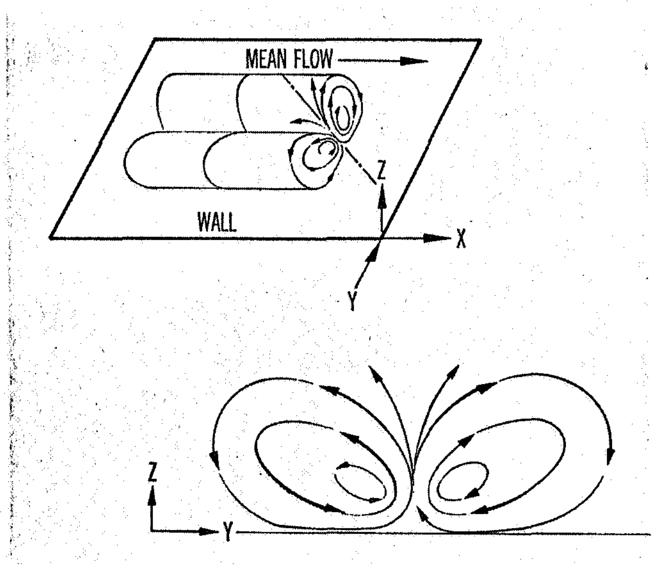
PODT	ORR
(EXTRACTION FROM EXPERIMENT)	(PREDICTION)

LARGE EDDY STRUCTURE OF FLOW PROTO-TYPE

1966 ⟵ 2-D WAKE ⟶ 1968 (NEXT SLIDE)

1966 ⟵ VISCOUS SUB-LAYER ⟶1967

1973-4 FLAT PLATE B.L. (PRELIMINARY TO NSG-2077)



ATTACHED (WALL) EDDIES CHARACTERISTIC OF TURBULENT BOUNDARY LAYERS (AFTER BAKEWELL,1966, AND ELSWICK, 1967)

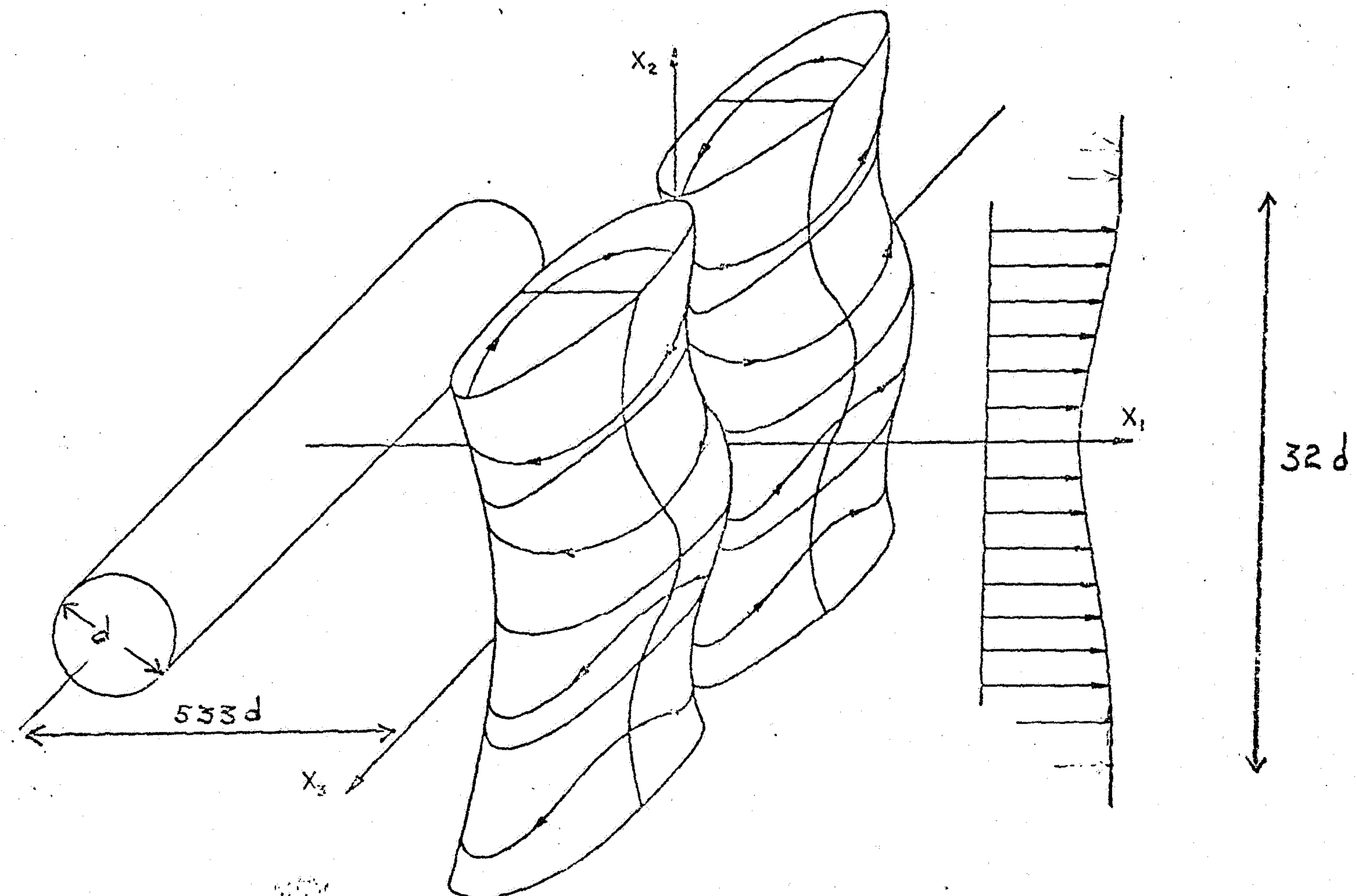


FIGURE 1 ARTIST'S CONCEPTION OF THE VORTEX PAIR (<u>NOT TO SCALE</u>). CURVED LINES WITH ARROWS ON SURFACE ARE STREAMLINES; NOTE TIP OF PLANES OF CIRCULATION NORMAL TO POSITIVE STRAIN RATE (PAYNE, 1966)

I. C. STATEMENT OF CURRENT PROBLEM (NSG-2077)

OBJECTIVE: To extract the "Large Eddy" structure from existing turbulent boundary layer Experimental Data

METHODOLOGY: Application of Lumley's proper Orthogonal Decomposition Theorem (PODT) to the empirical data of Grant (1959) merged with those of Tritton (1967). The work, as proposed and funded by NASA Grant NSG-2077, consists of four phases. A fifth phase, "Data Reduction Package" was proposed but no funding requested at this time. Each phase consists of tasks outlined below:

PHASE I: Construction of Fourier Transform of R_{ij}

- Task IA: Development of R_{ij} data grid
- Task IB: Construction of G_{ij} = F.T. (R_{ij})

PHASE II: Application of PODT to extract eigenvalues from G_{ij}

- Task IIA: Use of CM and symmetries to fill in G_{ij}
- Task IIB: Solution of (complex) eigenvalue problem in wave-number plane and y-space.

PHASE III: Construction of "Big Eddy" in "Real" Space

PHASE III:

TASK IIIA: INVERSE FOURIER TRANSFORM OF LARGEST EIGEN-FUNCTION, $\psi_i^{(1)}$

TASK IIIB: CONSTRUCTION OF VELOCITY FIELD $\underline{U}_e$ OF THE LARGEST EDDY (GRAPHICS)

TASK IIIC: INTERPRETATION OF LARGEST EDDY AS A 3-D STRUCTURE

PHASE IV: CONSTRUCTION OF B_{ij}, "BIG EDDY" COVARIANCE AND ν_e CALCULATION

TASK IVA: $B_{ij} = (U_e)_i \, (U_e)_j$

TASK IVB: $\nu_e = (B_{ij} - R_{ij}) / \frac{\partial U_i}{\partial X_j}$

TASK IVC: IS ν_e UNIFORM: WHY? WHY NOT? THIS IS A CRUCIAL TEST OF APPLICABILITY OF "EDDY VISCOSITY."

II. REVIEW OF PODT APPLIED TO BOUNDARY LAYER

<u>RECALL</u> PODT EXTRACTIVE METHOD:

A. EXPERIMENTAL DATA: $R_{ij} \, (\underline{X},T;\underline{X}',T') \equiv \langle u_i(\underline{X},T)u_j(X',T')\rangle$

B. SOLVE EIGENVALUE PROBLEM:

$$\int R_{ij} \, \phi_j^{(n)} d\underline{x}'dt' = \lambda^{(n)} \phi_i^{(n)}(\underline{x},t)$$

c. RECONSTRUCT $R_{ij} = \sum_{n=1}^{N} \lambda^{(n)} \phi_i^{(n)} \phi_j^{(n)*}$, N "SMALL"(=1 HERE)

ABOVE FOR A COMPLETELY GENERAL TURBULENT FLOW

II. A. SIMPLIFICATION DUE TO HOMOGENEITY OF STATISTICS

1) OUR B.L. ARE 2-D, SO X_3 IS AN EXACTLY HOMOGENEOUS DIRECTION

2) THE DATA ARE FROM FULLY-DEVELOPED BOUNDARY-LAYERS, $\therefore$ X_1 IS AN APPROXIMATELY HOMOGENEOUS DIRECTION

3) THE FLOWS ARE ALSO STATIONARY (HOMOGENEOUS IN TIME) AND WE DEAL NOT WITH ENSEMBLE BUT TEMPORAL AVERAGES:

$$3) \Rightarrow \quad R_{ij}(\underline{X},T;\underline{X}',T') \longrightarrow R_{ij}(\underline{X},\underline{X}')$$

$$1)\ \&\ 2) \Rightarrow R_{ij}(\underline{X};X') \longrightarrow R_{ij}(Y;Y',\underline{r})$$

WHERE $\underline{r} = (r_1, 0, r_3)$; $r_1 = X_1' - X_1$, $r_3 = X_3' - X_3$

SO, INSTEAD OF R_{ij} A FUNCTION OF 8 VARIABLES, WE HAVE ONLY 4.

LUMLEY (1965), SHOWS THAT THE ORIGINAL PODT - PROBLEM

$$\left(\int R_{ij}\ \phi_j^{(n)} d\underline{x}'dt' = \lambda^{(n)} \phi_i^{(n)}(\underline{x},t)\right)$$

REDUCES TO $\int G_{ij}(y,y';\underline{k})\psi_j^{(n)}(\underline{k},y')\ dy' = \lambda^{(n)}(\underline{k})\psi_i^{(n)}(\underline{k},y)$

WHERE $\underline{k} = (k_1, 0, k_3)$ IS WAVE NUMBER VECTOR IN HOMOGENEOUS DIRECTION AND

II. A. $G_{ij}(y,y';\underline{k}) = \text{F.T.}\ (R_{ij}(y;y',\underline{r})$

$$\phi_i^{(n)}(\underline{x}) = e^{i\underline{k}\cdot\underline{r}}\phi_i(\underline{k},y)$$

THESE RESULTS LUMLEY DENOTES BY HARMONIC ORTHOGONAL DECOMPOSITION THEOREM (HODT); IN WORDS:

HODT => IN ANY HOMOGENEOUS DIRECTION*, THE HARMONIC FUNCTIONS ARE THE EIGENFUNCTIONS.

FURTHERMORE: HODT => 6 INDEPENDENT VARIABLES $(\underline{x},\underline{x}')$ ARE COLLAPSED INTO 4 $(y,y',\underline{r})$ AND BY FOURIER TRANSFORMING THE λ-VALUE PROBLEM INTO 2 VARIABLES (y,y') ON A PARAMETRIC, 2-D $\underline{k}$-GRID.

A GIGANTIC SAVINGS IN TIME & $!

NOTE: FOR THIS SAVINGS, WE DO PAY A SLIGHT PENALTY; NAMELY:

HOMOGENEOUS => THERE IS NO FINITE SCALE IN THAT DIRECTION, SO "LARGE EDDY" CONSTRUCTION IN $\underline{X}$-SPACE IS MORE INVOLVED

---

*REQUIRED TO DEFINE A FOURIER TRANSFORM

B. IDENTIFICATION OF THE BIG EDDIES

AGAIN, LUMLEY (1965):

$$(Ue)_i = \iint_{-\infty}^{\infty} e^{i\underline{k}\cdot\underline{x}} \sqrt{\frac{\lambda^{(1)}}{2\pi}}\, \psi_i^{(1)}(\underline{k},y)\,d\underline{k} \tag{B1}$$

WHERE $(Ue)_i$ IS THE "BIG EDDY" IN $\underline{X} = (X,Y,Z)=(X_1,X_2,X_3)$ - SPACE, WHICH IS JUSTIFIED BY

1. IT IS A RATIONAL DEFINITION OF $(Ue)_i$
2. IF THE SPECTRAL PEAK OF $\lambda^{(1)}$ (IN $\underline{K}$-SPACE) IS "SHARP," THEN $(Ue)_i$ IS "ALL" OF THE LARGE STRUCTURE.
3. IF THE PEAK IN $\lambda^{(1)}$ IS NOT SHARP, THEN $(Ue)_i$ HAS CORRESPONDINGLY LESS INFORMATION ABOUT THE "REAL" LARGEST STRUCTURE.

HOWEVER, IN ALL CASES (OF $\lambda^{(1)}$ SPECTRAL "SHARPNESS") THE Y-VARIATION OF $(Ue)_i$ IS UNAFFECTED.

REMARK: EQ. (B1) ABOVE, AS WELL AS LUMLEY'S GENERAL CONTENTION THAT THE $\phi_i^{(n)}, \psi_i^{(n)}$ (n"SMALL") ARE LARGE EDDIES WAS FIRST VERIFIED BY PAYNE (1966) AND BAKEWELL (1966).

II. C. RECONSTITUTION OF 3-D "BIG EDDY" STRUCTURE

1) RECALL:

SOLUTION OF F.T. PODT (+HODT) EIGENVALUE PROBLEM:

$$\int G_{ij}\, \psi_j^{(n)} dy' = \lambda^{(n)}(\underline{k})\, \psi^{(n)}(k,y) \tag{C1}$$

On a (parametric) 2-D $\underline{K}$-space grid, yields (when converted to finite mathematics)

$$\left\{ \begin{array}{l} \lambda^{(n)}(\underline{K}) \\ \psi_i^{(n)}(\underline{K}, y) \end{array} \right\} \quad \text{for } n = 1,2,---N \quad \underline{K} = (K_1, 0, K_3) \tag{C2}$$

where $\lambda^{(n)}$ are the mean square energies in the $\psi_i^{(n)}$ eigen-mode ("structure" in wave number space)

2) Eq. (B1), the Rice Theorem - Lumley definition:

$$(Ue)_i = \iint_{-\infty}^{\infty} e^{i\underline{k}\cdot\underline{x}} \sqrt{\frac{\lambda^{(1)}}{2\pi}}\, \psi_i^{(1)}(\underline{k}, y)\, d\underline{k} \tag{C3}$$

yields a representative, "statistically persistent," structure for the largest "eddy."

3) <u>For Wall Layer</u>, Bakewell's (1966) experiments yielded N = 5 eigenmodes, of which only the first had amplitude greater than the "noise" level of empiric error, <u>For 2-D Wake</u>, Grant's (1959) data yielded N = 15 modes but the second had about 50% experiment error and was not interpreted (Payne 1966)

4) For NSG-2077, we anticipate N ≅ 15-24 eigenmodes, only the first (ala Bakewell) will, probably, yield usable results. The grant will inverse Fourier Transform $\sqrt{\lambda^{(1)}}\psi_i^{(1)}$ to get

$$\underline{U}_e(\underline{X}) = \text{F.T.}\left(\sqrt{\lambda^{(1)}}\,\psi^{(1)}\right) \text{ on } \underline{X} = (X_1, X_2, X_3)\text{-Grid} \tag{C4}$$

5) THE 3-D VECTOR, $\underline{U}_e$, WILL BE GRAPHICALLY DISPLAYED AND USED TO CONSTRUCT A 3-D EDDY STRUCTURE (ANALOGOUS TO SLIDES 14 & 15 FOR WALL EDDY AND WAKE VORTEX PAIR + "RE-ENTRANT" JETS)

6) THE LAST PHASE (VI) WILL, FROM EQ. (C-4), FORM:

$$B_{ij}(\underline{X};\underline{X}') = (U_e(\underline{X})_i\ (U_e(\underline{X}')_j) \tag{C5}$$

THE BIG EDDY CO-VARIANCE TENSOR, AND

$$B_{ij}(\underline{X}) = B_{ij}(\underline{X},\underline{X}) \tag{C5}$$

THE BIG EDDY "REYNOLD'S STRESS" AT POINT $\underline{X}$ ALSO,

$$\nu_e(\underline{X}) = [B_{ij}(\underline{X}) - R_{ij}(\underline{X})]/\frac{\partial U_i}{\partial X_j} \tag{C6}$$

WILL BE CALCULATED. THE CONJECTURE TO BE TESTED IS:

"IS ν_e ISOTROPIC (OR HOMOGENEOUS IN X_1,X_3)?"

II.D. RECAP OF SEQUENTIAL STEPS (NSG-2077)

1) CONSTRUCTION OF G_{ij}= F.T.(R_{ij})
   - A) MERGER OF EMPIRIC DATA SETS (GRANT, TRITTON)
   - B) FILL IN DATA VOIDS OF R_{ij} BY SYMMETRIES AND "RELAXATION" METHODS
   - C) FOURIER TRANSFORM 6 R_{ij}-COMPONENTS

2) SOLVE PODT EIGEN-VALUE PROBLEM
   - A) FILL IN REMAINING 3 COMPONENTS OF G_{ij} BY FLUID CONTINUITY:

$$\frac{\partial R_{ij}}{\partial X_i} = 0 = \frac{\partial R_{ij}}{\partial X'_j} \Rightarrow \begin{cases} ik_p\, G_{pj} + \dfrac{\partial}{\partial r_2} G_{2j} = 0 \\ ik_p\, G_{jp} + \dfrac{\partial}{\partial r_2} G_{j2} = 0 \end{cases} \begin{cases} j = 1,2,3 \\ p = 1,3 \end{cases} \tag{D-1}$$

B) CONVERT FROM INTEGRAL TO MATRIX PROBLEM (FINITE MATH)

$$\underline{\text{PODT}}: \int_{-\infty}^{\infty} G_{ij}(Y,Y')\psi_j^{(n)}(Y')dY' = \lambda^{(n)}(\underline{k})\psi_i^{(n)}(\underline{K},Y) \qquad \text{(D-2)}$$

WHERE $i,j = 1,2,3;\ n = 1,2,3,$ ----. EQ. (D-2) IN MATRIX FORM:

$$\int_{-\infty}^{\infty} \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{21} \\ G_{31} & G_{32} & G_{33} \end{pmatrix} \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix} dY' = \lambda^{(n)} \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix} \qquad \text{(D-3)}$$

NOTE: 1) THE WALL IS $Y' = 0$, SO LOWER LIMIT $\rightarrow 0$

2) ANTICIPATE USING 5 TO 8 Y' VALUES; SAY, 5.

3) FOR $Y' > \delta$, PRESUMABLY R_{ij} (& G_{ij}) $\rightarrow 0$

HENCE, WE CAN APPROXIMATE THE FINITE INTEGRAL

$$\int_0^{\delta} G_{ij}\, \psi_j^{(n)} dY' = \lambda^n \psi_i^n\ (Y)$$

BY

$$\sum_{e=1}^{5} G_{ij}(Y_K, Y'_e)\psi_j^{(n)} \Delta Y'_e = \lambda^{(n)} \psi_i^{(n)}(yK) \qquad \text{(D-4)}$$

NOTE: THIS METHOD, ACCORDING TO WIELANDT (1956) AND AS VERIFIED BY PAYNE (1966), YIELDS A MAXIMUM ERROR OF ORDER 1-2%, WELL WITHIN EXPERIMENTAL ERROR! (TO OBTAIN THIS PRECISION SOME CARE IS NEEDED -- SEE (PAYNE, 1966), P. 21 AND APPENDIX D FOR CRITERIA, SCALINGS, AND CHECKS). IN MATRIX FORM, EQ. D-4 FOR EQUAL YK,Ye SPACING IS:

$$\begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix} \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix} = \lambda^{(n)} \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix} \tag{D-5}$$

WHERE THE EQUAL $\Delta Y_e'$ ARE ABSORBED INTO G_{ij} AND EACH G_{ij} IN THE SQUARE ARRAY ARE THEMSELVES 5 X 5 MATRICES, I.E.,

$$G_{11} = \begin{pmatrix} G_{11}(Y_1,Y_1') & G_{11}(Y_1,Y_2') & G_{11}(Y_1,Y_3') & G_{11}(Y_1,Y_4') & G_{11}(Y_1,Y_5') \\ G_{11}(Y_2,Y_1') & G_{11}(Y_2,Y_2') & G_{11}(Y_2,Y_3') & G_{11}(Y_2,Y_4') & G_{11}(Y_2,Y_5') \\ G_{11}(Y_3,Y_1') & G_{11}(Y_3,Y_2') & G_{11}(Y_3,Y_3') & G_{11}(Y_3,Y_4') & G_{11}(Y_3,Y_5') \\ G_{11}(Y_4,Y_1') & G_{11}(Y_4,Y_2') & G_{11}(Y_4,Y_3') & G_{11}(Y_4,Y_4') & G_{11}(Y_4,Y_5') \\ G_{11}(Y_5,Y_1') & G_{11}(Y_5,Y_2') & G_{11}(Y_5,Y_3') & G_{11}(Y_5,Y_4') & G_{11}(Y_5,Y_5') \end{pmatrix}$$

AND EACH $\psi_1^{(n)}$ IS ITSELF A COLUMN 5-VECTOR:

ON LHS:

$$\psi_1^{(n)} = \begin{pmatrix} \psi_1^{(n)}(y_1') \\ \psi_1^{(n)}(y_2') \\ \psi_1^{(n)}(y_3') \\ \psi_1^{(n)}(y_4') \\ \psi_1^{(n)}(y_5') \end{pmatrix} \quad \text{AND ON RHS:} \quad \psi_1^{(n)} = \begin{pmatrix} \psi_1^{(n)}(y_1) \\ \psi_1^{(n)}(y_2) \\ \psi_1^{(n)}(y_3) \\ \psi_1^{(n)}(y_4) \\ \psi_1^{(n)}(y_5) \end{pmatrix}$$

NOTE: THE CONVERSION TO MATRIX FORM NOW MEANS THAT (FOR N = 5) WE NOW HAVE EXPANDED FROM A 3 X 3 ARRAY (G_{ij} IN EQ. D-3) TO A 15 X 15 ARRAY OF COMPLEX ENTRIES AND 15-COMPONENT $\underline{\psi}$ VECTORS (FROM ORIGINAL 3-VECTORS)

c) SOLUTION OF EIGEN VALUE PROBLEM (EQ. D5):

SINCE D5 INVOLVES A COMPLEX 15 X 15 MATRIX AND G IS HERMITIAN, THE EASY WAY TO SOLVE IS TO CONVERT TO A REAL 30 X 30 MATRIX WHICH IS SYMMETRIC.

HERMITIAN => $G_{ij}(Y,Y') = G^{*}_{ji}(Y',Y)$ (D-6)

OR, DECOMPOSING G_{ij} INTO REAL, C_{ij} , AND IMAGINARY, Q_{ij}, PARTS:

$$G_{ij} = C_{ij} + i\, Q_{ij} \tag{D-7}$$

THEN D-4 BECOMES

$$\sum_{e=1}^{5} \begin{pmatrix} C_{ij} & -Q_{ij} \\ Q_{ij} & C_{ij} \end{pmatrix} \begin{pmatrix} X_j^{(n)} \\ Y_j^{(n)} \end{pmatrix} = \lambda^{(n)} \begin{pmatrix} X_i^{(n)} \\ Y_i^{(n)} \end{pmatrix} \tag{D-8}$$

WHERE $\psi_j^{(n)} = X_j^{(n)} + i\, Y_j^{(n)}$, $n = 1,2,\text{---}\ 15$ (D-9)

SO, OF COURSE, THE $\lambda^{(n)}$ OCCUR IN PAIRS ASSOCIATED WITH $\psi_j^{(n)}$ COMPLEX PAIRS.

NOTE: THIS IS NOT REALLY AN ADDED COMPLEXITY SINCE COMPLEX EIGENVECTORS ARE INHERENTLY UNIQUE ONLY TO WITHIN AN ARBITRARY PHASE ANGLE WHEN NORMALIZED.

3) CONSTRUCTION OF "BIG EDDY"

HAVING SOLVED FOR $\lambda^{(1)}$, $\psi^{(1)}(\underline{K},Y)$, THE LARGEST AMPLITUDE EIGENMODE, AN INVERSE F.T. YIELDS

$$(U_e)_i = \iint_{-\infty}^{\infty} e^{i\underline{K}\cdot\underline{X}} \sqrt{\frac{\lambda^{(1)}}{\pi}}\ \psi_i^{(1)}(\underline{K},Y)\, d\underline{K} \tag{D-10}$$

COMPUTER GRAPHICS OF D-10 VECTOR COMPONENTS WILL PERMIT CONSTRUCTION OF THE 3-D STRUCTURE

4) CONSTRUCTION OF B_{ij}, BIG EDDY CO-VARIANCE

$$\text{D-10} \Rightarrow B_{ij} = (U_e)_i(U_e)_j \tag{D-11}$$

AND $$\nu_e = (B_{ij} - R_{ij})/\frac{\partial U}{\partial Y} \tag{D-12}$$

S U M M A R Y

1. EXPERIMENTAL $R_{ij} = \overline{U_i U_j}$

   G_{ij} = F.T. (R_{ij}) IN HOMOGENEOUS DIRECTIONS

2. SOLVE PODT EIGEN-VALUE PROBLEM

   $$G_{ij}\ \psi_j^{(n)} = \lambda^{(n)}\psi_i^{(n)}$$

3. $(U_e)_i$ = F.T. $(\sqrt{\lambda^{(1)}}\psi_i^{(1)})$

4. $B_{ij} = (U_e)_i\ (U_e)_j$

   $$\nu_e = (B_{ij} - R_{ij})/\frac{\partial U}{\partial Y}$$

TASKS OF STUDY

I CORRELATION DATA DEVELOPMENT

- SOURCE IDENTIFICATION
- DATA AUGMENTATION
- DATA DENORMALIZATION
- DATA BASE ESTABLISHMENT

II FOURIER TRANSFORM CALCULATION

- ALGORITHM IDENTIFICATION
- DATA BASE ESTABLISHMENT

III EIGENVALUE PROBLEM SOLUTION

- ALGORITHM IDENTIFICATION
- INTERPRETATION OF EIGENMODES
- DATA BASE ESTABLISHMENT

IV VELOCITY FIELD RECONSTRUCTION

- INVERSE FOURIER TRANSFORM CALCULATION
- ν_e ISOTROPY DETERMINATION
- DOMINANT MOTION REPRESENTATION

I CORRELATION DATA DEVELOPMENT

- SOURCE IDENTIFICATION
  - GRANT (JFM, 1959)
  - TRITTON (JFM, 1967)
  - TOWNSEND (MONOGRAPH, 1956)

- DATA AUGMENTATION
  - INTERPOLATION
  - CONTINUITY
  - SYMMETRY

- DATA DENORMALIZATION
  - R_{ij} NORMALIZATION PROCEDURE?
  - TURBULENCE INTENSITY DATA SOURCE

SUMMARY OF AVAILABLE DATA

R_{ij} vs r_1

$i \backslash j$	1	2	3
1	X	X	
2	X	X	
3			X

R_{ij} vs r_2

$i \backslash j$	1	2	3
1	X	X	
2		X	
3			X

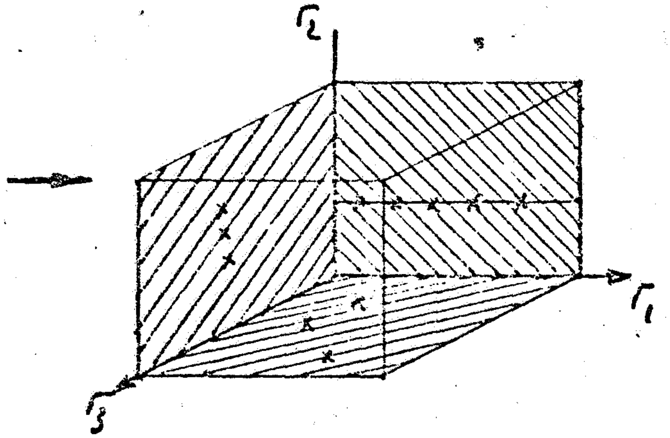
R_{ij} vs r_3

$i \backslash j$	1	2	3
1	X	X	
2		X	
3			X

- DATA AUGMENTATION - INTERPOLATION

  - AT CONSTANT FIXED PROBE ALTITUDE - LAPLACES EQUATION
    (INTERPOLATION IN r_1, r_2, r_3 GRID)

BOUNDARY PLANES

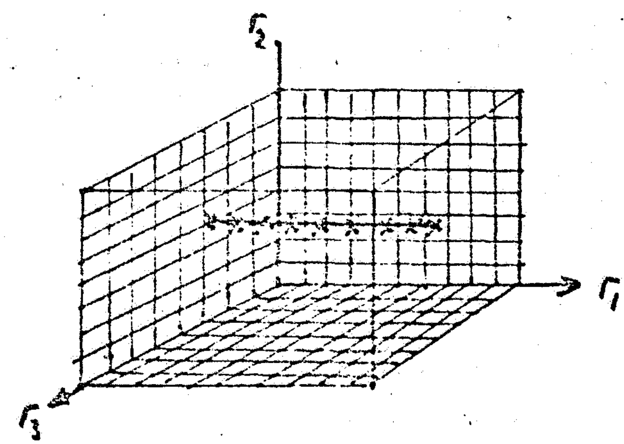


$$\frac{\partial^2 R}{\partial x_1^2} + \frac{\partial^2 R}{\partial x_2^2} = F$$

$$\begin{aligned} R(x_1, 0) &= f_1(x) \\ R(0, x_2) &= f_2(x) \\ R(x_1, U.L.) &= 0 \\ R(U.L., x_2) &= 0 \end{aligned}$$

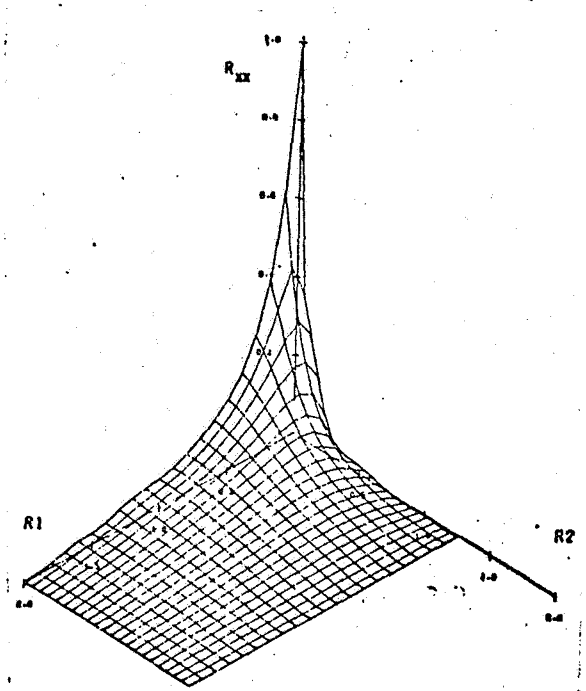
SOLUTION : $R = g(x_1, x_2)$

FIELD VALUES

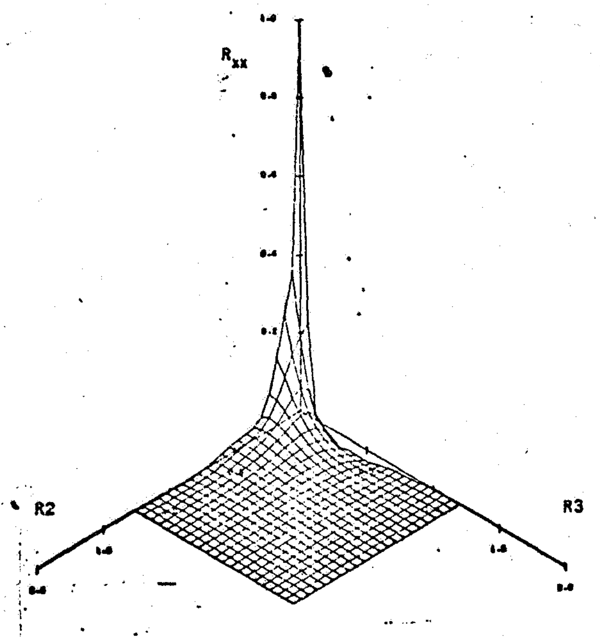


$$\frac{\partial^2 R}{\partial x_1^2} + \frac{\partial^2 R}{\partial x_2^2} + \frac{\partial^2 R}{\partial x_3^2} = F$$

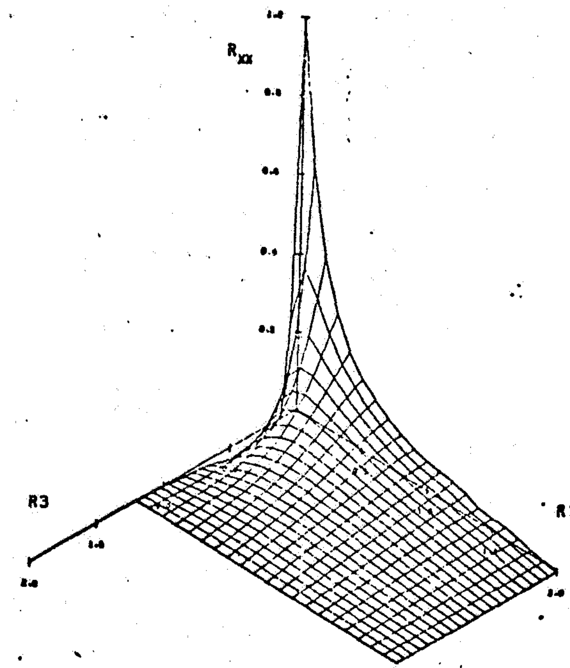
$$\begin{aligned} R(x_1, x_2, 0) &= g_1(x_1, x_2) \\ R(0, x_2, x_3) &= g_2(x_2, x_3) \\ R(x_1, 0, x_3) &= g_3(x_1, x_3) \\ R(x_1, x_2, U.L.) &= 0 \\ R(UL, x_2, x_3) &= 0 \\ R(x_1, UL, x_3) &= 0 \end{aligned}$$



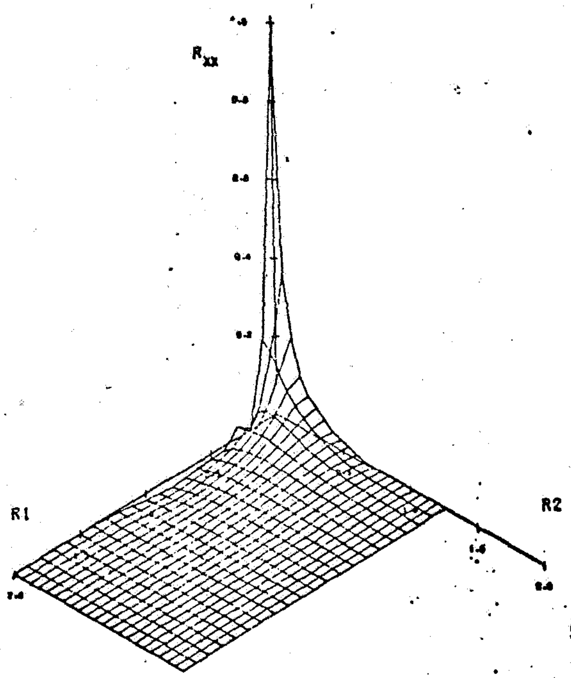
CORRELATION DATA AUGMENTATION
R11 BOUNDARY VALUES - GRANT DATA
YF = .05



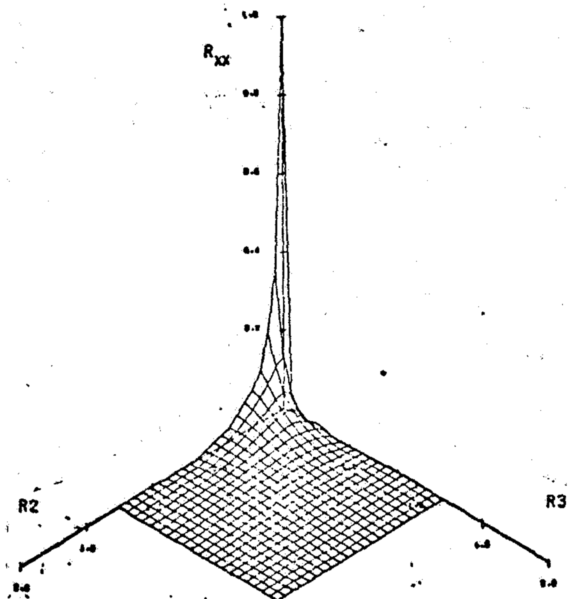
CORRELATION DATA AUGMENTATION
R11 BOUNDARY VALUES - GRANT DATA
YF = .05



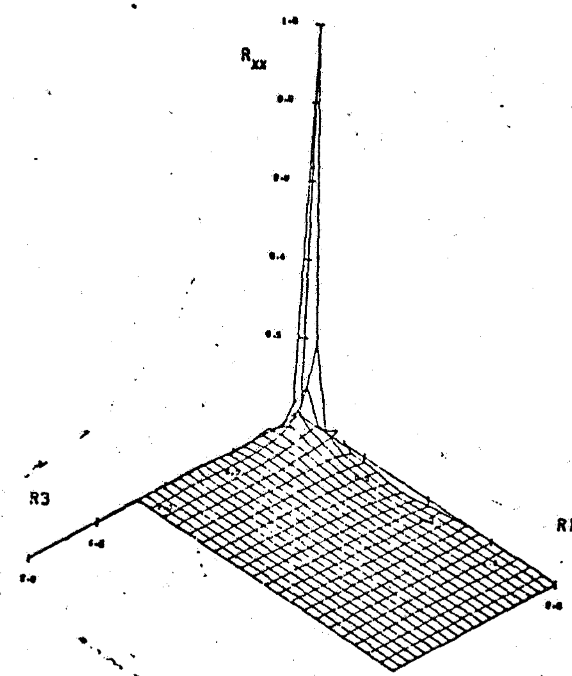
CORRELATION DATA AUGMENTATION
R11 BOUNDARY VALUES - GRANT DATA
YF = .05



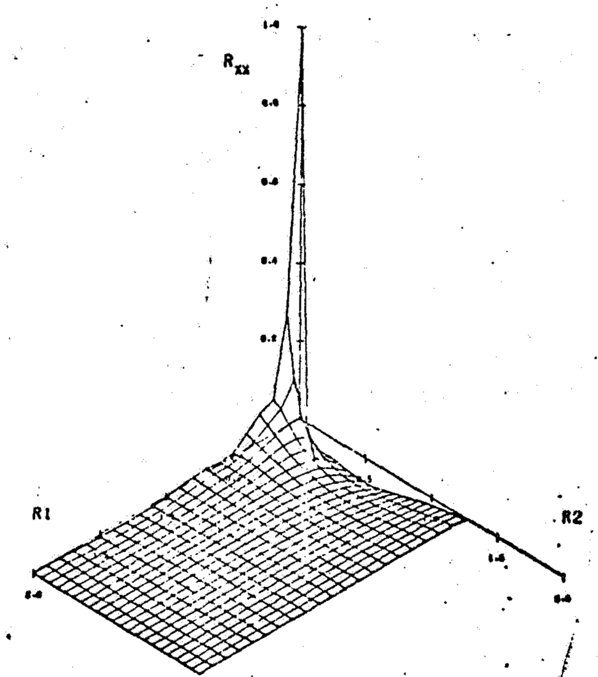
CORRELATION DATA AUGMENTATION
R22 BOUNDARY VALUES - GRANT DATA
YF = .05



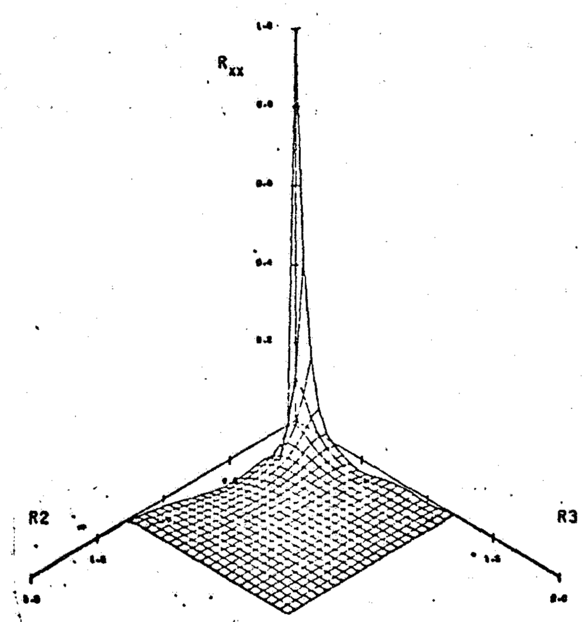
CORRELATION DATA AUGMENTATION
R22 BOUNDARY VALUES - GRANT DATA
YF = .05



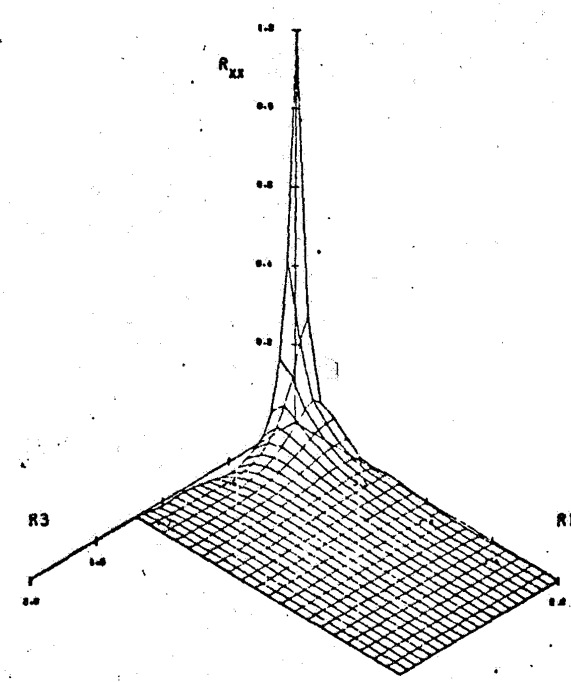
CORRELATION DATA AUGMENTATION
R22 BOUNDARY VALUES - GRANT DATA
YF = .05



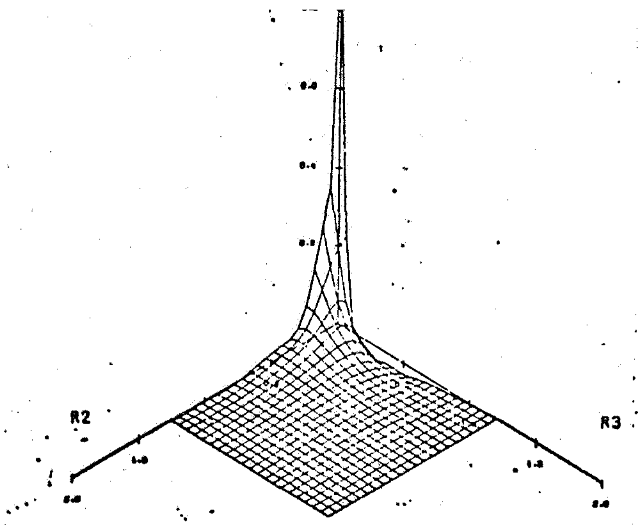
CORRELATION DATA AUGMENTATION
R33 BOUNDARY VALUES - GRANT DATA
YF = .05



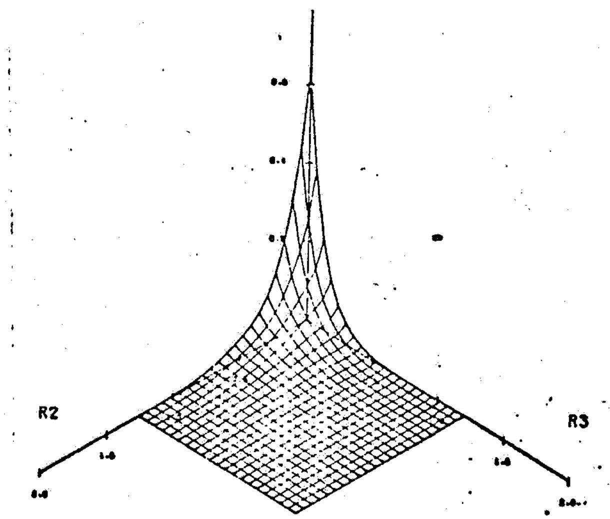
CORRELATION DATA AUGMENTATION
R33 BOUNDARY VALUES - GRANT DATA
YF = .05



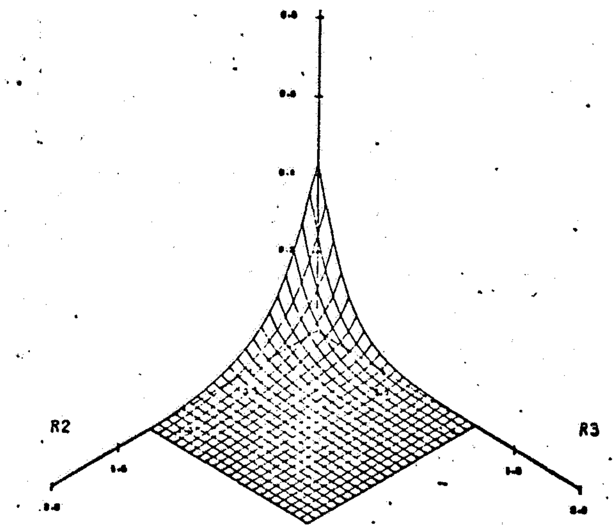
CORRELATION DATA AUGMENTATION
R33 BOUNDARY VALUES - GRANT DATA
YF = .05



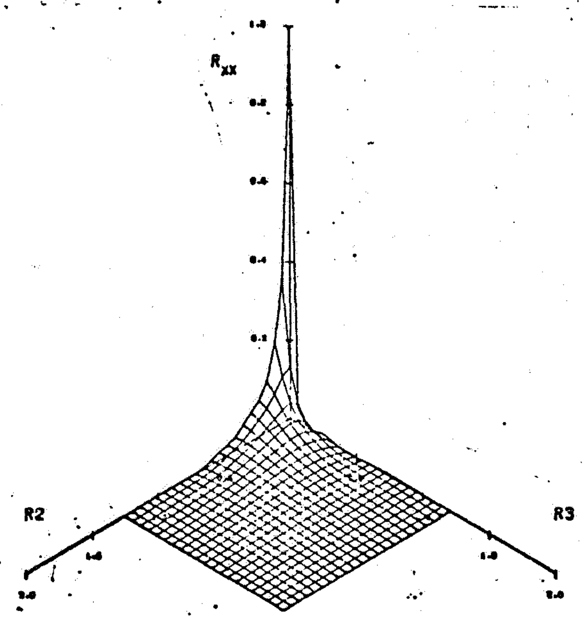
CORRELATION DATA AUGMENTATION
R11 FIELD VALUES - GRANT DATA
YF = .05 R1 = .0



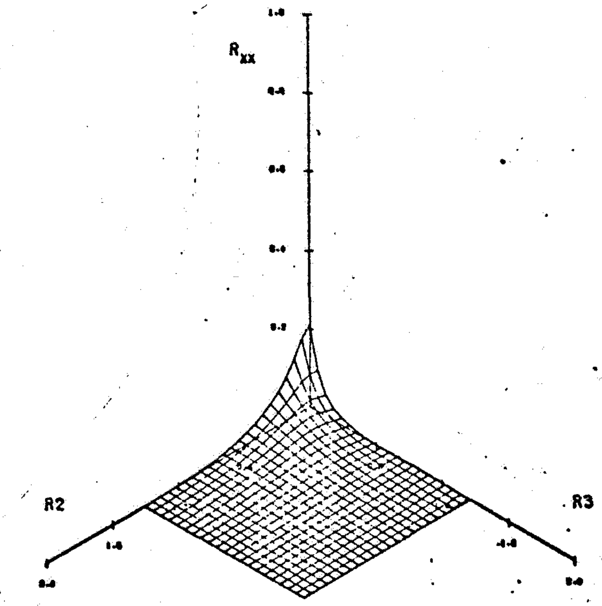
CORRELATION DATA AUGMENTATION
R11 FIELD VALUES - GRANT DATA
YF = .05 R1 = .1



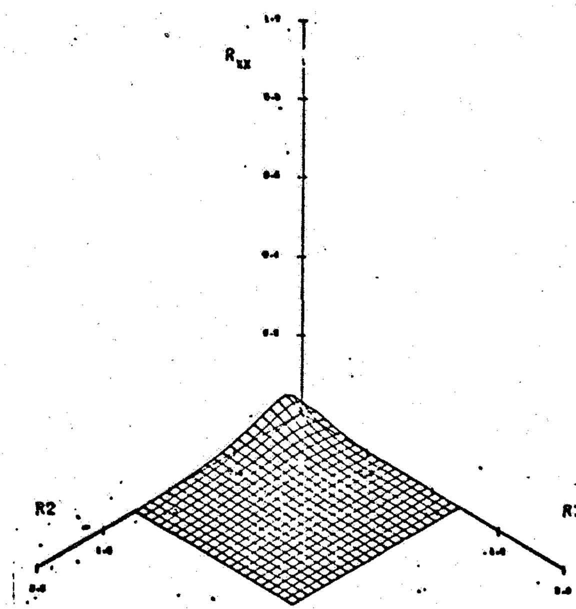
CORRELATION DATA AUGMENTATION
R11 FIELD VALUES - GRANT DATA
YF = .05 R1 = .2



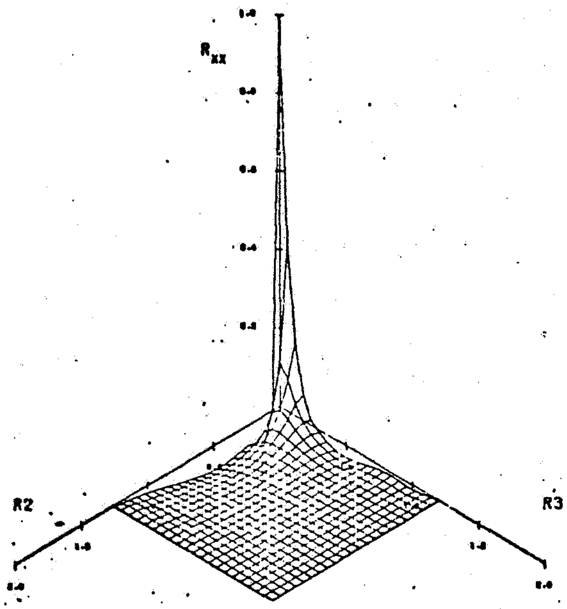
CORRELATION DATA AUGMENTATION



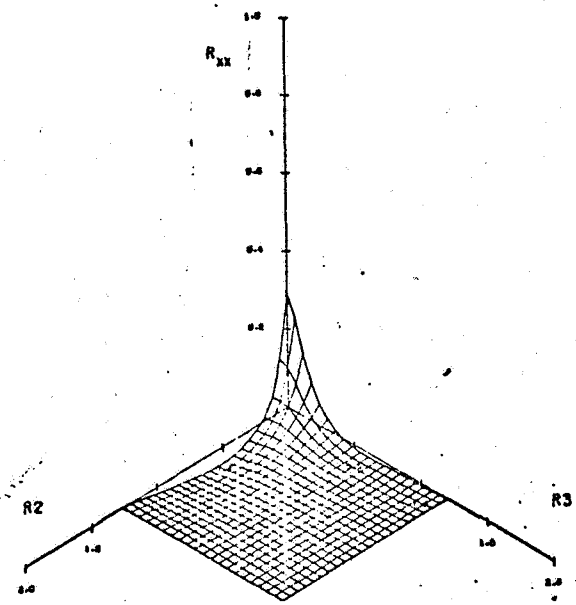
CORRELATION DATA AUGMENTATION
R22 FIELD VALUES - GRANT DATA



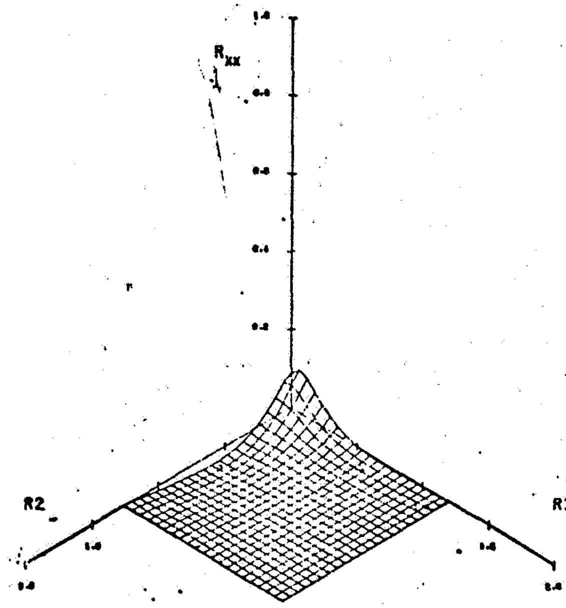
CORRELATION DATA AUGMENTATION
R22 FIELD VALUES - GRANT DATA



CORRELATION DATA AUGMENTATION
R33 FIELD VALUES - GRANT DATA
YF = .05 R1 = .0



CORRELATION DATA AUGMENTATION
R33 FIELD VALUES - GRANT DATA
YF = .05 R1 = .1



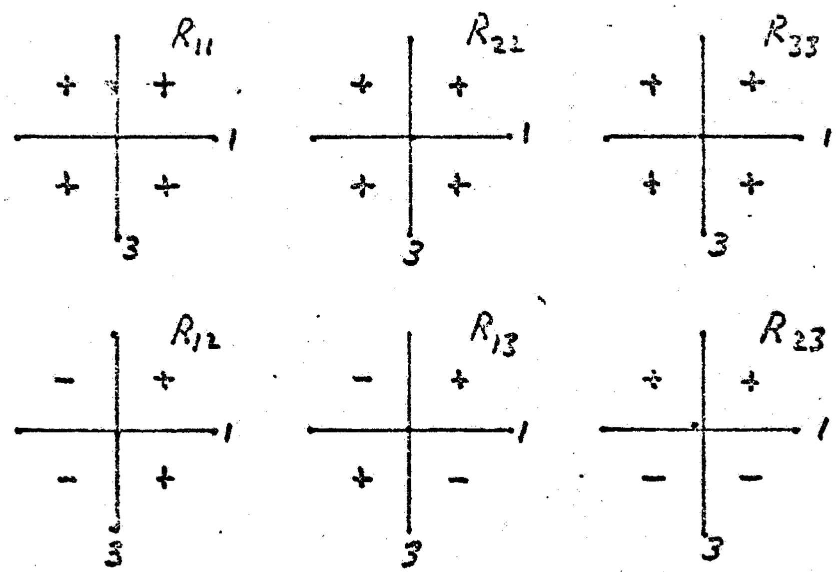
CORRELATION DATA AUGMENTATION
R33 FIELD VALUES - GRANT DATA
YF = .05 R1 = .2

INTERPOLATION

- AT VARIOUS FIXED PROBE ALTITUDES (Y_F)
  - GENERATES REGULAR DATA BLOCK FOR REGULAR Y_F SPACING
  - USES LINEAR OR PARABOLIC FIT, POINT BY POINT

SYMMETRY

- SYMMETRY PROPERTIES IN 1-3 PLANE



- RESULTING CORE REQUIREMENTS : 1/6 OF FULL ARRAY

CONTINUITY

CONSERVATION OF MASS (INCOMPRESSIBLE $\frac{\partial u_i}{\partial x_i} = 0$)

APPLIED TO THE CORRELATION TENSOR R_{ij} $(= u_i(\bar{x})\, u_j(\bar{x}'))$

GIVES

$$\frac{\partial}{\partial x_i} R_{ij}(\bar{x}, \bar{x}') = 0 = \frac{\partial}{\partial x_j} R_{ij}(\bar{x}, \bar{x}')$$

OR

$$\frac{\partial}{\partial x_i}\left[u_i(\bar{x})\, u_j(\bar{x}')\right] = 0 = \frac{\partial}{\partial x_j}\left[u_i(\bar{x})\, u_j(\bar{x}')\right]$$

$$u_j(\bar{x}')\frac{\partial u_1}{\partial x_1}(\bar{x}) + u_j(\bar{x}')\frac{\partial u_2}{\partial x_2}(\bar{x}) + u_j(\bar{x}')\frac{\partial u_3}{\partial x_3}(\bar{x}) = 0$$

ALTERNATE APPLICATION:

IN WAVE NUMBER SPACE, THE ABOVE CONDITIONS BECOME

$$i\, k_i G_{ij} + \frac{d}{dx_2} G_{2j} = 0$$

$$-i\, k_i G_{ji} + \frac{d}{dx_2'} G_{j2} = 0$$

DATA DENORMALIZATION

TWO METHODS OF NORMALIZATION

- $$R_{ij}(\bar{x}, \bar{x}') = \frac{\overline{u_i(\bar{x})\, u_j(\bar{x}')}}{\left[\overline{u_i^2}(\bar{x})\, \overline{u_j^2}(\bar{x})\right]^{1/2}}$$

- $$R_{ij}(\bar{x}, \bar{x}') = \frac{\overline{u_i(\bar{x})\, u_j(\bar{x}')}}{\left[\overline{u_i^2}(\bar{x})\, \overline{u_j^2}(\bar{x}')\right]^{1/2}}$$

DATA SOURCE - TOWNSEND

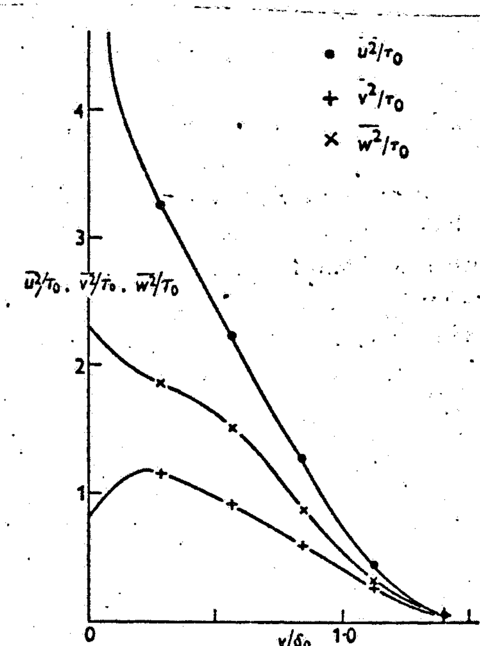


Fig. 10.8. Distribution of turbulent intensities in a boundary layer (after Klebanoff, 1954).

II FOURIER TRANSFORM

TRANSFORM TO BE PERFORMED IS

$$G^{*} = \iint_{-\infty}^{\infty} R e^{-i(k_1 r_1 + k_3 r_3)} \, dr_1 \, dr_3$$

REPRESENTATION OF ABOVE FORMULA IS

$$G = \sum_{-r_{3\,Limit}}^{r_{3\,Limit}} \sum_{-r_1}^{r_1} R e^{-i(k_1 r_1 + k_3 r_3)} \Delta r_1 \Delta r_3$$

WHERE $\Delta r_1 = f(k_1)$
$\Delta r_3 = f(k_3)$

Δr_1 & Δr_3 ARE CHOSEN TO PROVIDE AT LEAST 10 INTERVALS IN ONE CYCLE OF $\cos kx$ CURVE.

i.e. $\Delta r = \dfrac{.1}{k}$

III EIGENVALUE PROBLEM SOLUTION

- ALGORITHM IDENTIFICATION

  COMPLEX SYSTEM OF EQUATIONS TO BE SOLVED

  - DIRECT SOLUTION OF COMPLEX EQUATIONS
  - SEPARATE SOLUTIONS FOR REAL AND IMAGINARY PARTS - RELIANCE ON HERMITIAN PROPERTIES

- INTERPRETATION OF EIGENMODES

IV VELOCITY FIELD RECONSTRUCTION

- INVERSE FOURIER TRANSFORM CALCULATION

$$(u_e)_i = \iint_{-\infty}^{\infty} e^{i\bar{k}\bar{x}} \sqrt{\frac{\lambda^{(1)}}{2\pi}}\, \varphi_i^{(1)}(\bar{k}, y)\, d\bar{k}$$

- ν_e ISOTROPY DETERMINATION

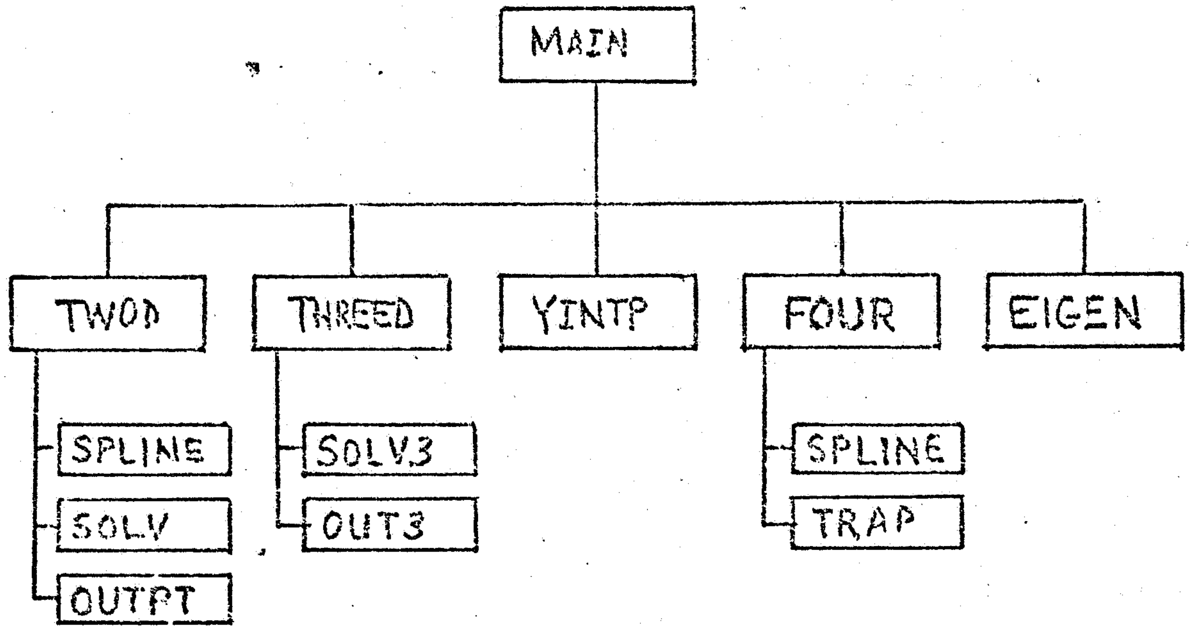
$$\nu_e = \left[\frac{\partial U_i}{\partial x_j}\right]^{-1} \left[B_{ij} - R_{ij}\right]$$

Where $B_{ij} = (u_e)_i (u_e)_j$, DOMINANT MODE

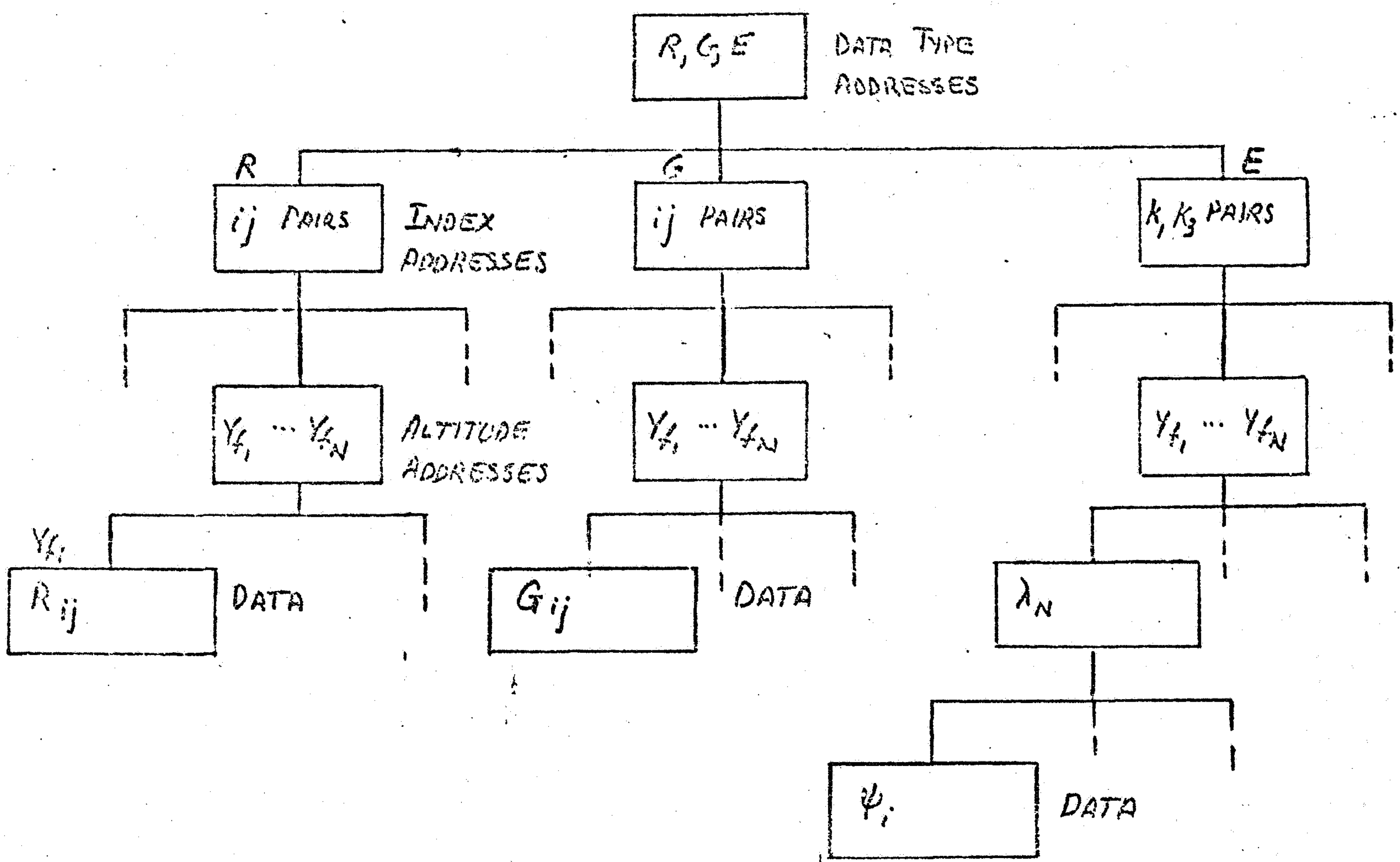
- DOMINANT MOTION REPRESENTATION

  - PLOTTING RESULTS OF INVERSE FOURIER TRANSFORM

COMPUTER ROUTINE STRUCTURE



DATA BASE STRUCTURE
(RANDOM ACCESS MASS STORAGE)



IV. Summary Status of Results and Budget

A. Results

	% Effort*	% Complete
I. Correlation Data Base	50	x70 = 35
II. Fourier Transform	30	x30 = 9
III. Eigen-value problem	15	**
IV. Big Eddy Construction	15	**
	100 %	~ 45 %

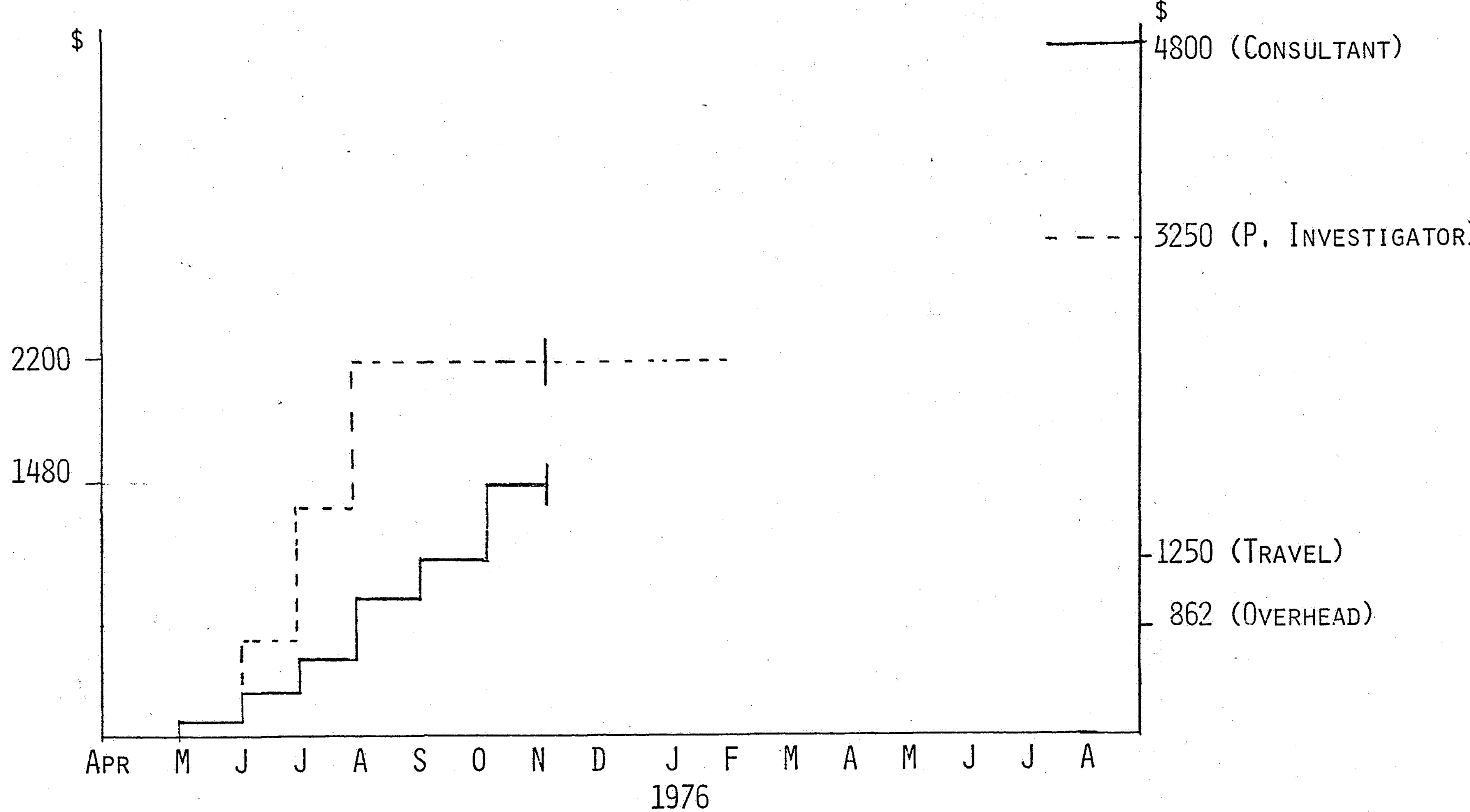
B. Budget (as of 31 October 1975)

Item	Granted	Expanded	Remaining
Principal Investigator	$ 3250.00	$ 2196.14	$1053.86
Consultant	4800.00	1482.50	3317.50
Travel	1250.00	-0-	1250.00
UTA Overhead	852.00	602.00	260.00
	$10,162.00	$4,280.64	$5,881.36

* % Effort = Relative to Grant total

** Denotes Planning Completed

IV. C. CHARGABLE TO NSG - 2077 BUDGET PHASING



- - - - - - PRINCIPAL INVESTIGATOR

———— CONSULTANT

EXPENDED THROUGH 31 OCTOBER 1975

V. POST NSG-2077 - WHAT REMAINS?

A. SHORT TERM:

1. AUTOMATED DATA ANALYSIS PROGRAM

2. PREDICTION (VIA "ORR") OF EIGEN-MODES OF TURBULENT PROFILE IN FLAT-PLATE B.L. (WITH NSG-2077 AS A "CONTROL")

3. IF NSG-2077 "EDDY VISCOSITY" IS ISOTROPIC, THEN MODEL REYNOLDS' EQS.

B. LONG TERM:

1. COMPREHENSIVE, 2-D B.L. ON FLAT PLATE

2. PRESSURE GRADIENT STRUCTURE EFFECTS

3. COMPRESSIBILITY EFFECTS

F I N A L E

SINCE PODT IS A STRUCTURE (NOT DYNAMICS) ORIENTED METHOD ITS APPLICABILITY IS UNIVERSAL.

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